**Unit 12: Eigentheory Applied to Linear Systems**

**Page 12.1**

Goals/Rationale

The students will develop some understanding of the idea of equilibrium solutions for linear systems. With the investigation on Page 12.1, the students grow to understand that linear systems either have one equilibrium solution at the point (0,0) in a phase plane, or an infinite number of equilibrium solution that lie on line(s) in the phase plane.

Implementation Notes and Student Thinking

For those who take linear algebra- this sets up the connection to the invertible matrix theory and so the instructor will likely see the students making these connections. If they don’t, the instructor can bring it up.

*Problem 1* – There are two possibilities for number of equilibrium solutions for autonomous linear systems of differential equations. Either (0,0) is the only equilibrium point (as in 1(a)), or there are an infinite number of equilibriums (as in 1(b)).

*Problem 2* – The expectation is that students will argue that there is either only one equilibrium solution (0,0) or there are infinitely many for systems of this form.Infinite number of equilibrium solutions occur when the equations ax + by = 0 and cx + dy = 0 are dependent.

*Problem 3* – This is the main result, with set up by problems 1 and 2. We expect students to work either geometrically (nullclines have to have the same slope, so –a/b = -c/d) or algebraically to come to equivalent conclusions about the relationship between a, b, c, d.  **After students have developed their criteria, the teacher should make the formal connection between students’ criteria and the determinant for the corresponding matrix.** To do this, the teacher should introduce matrix notation at this point. Setting the determinant equal to zero is intended to be a generalization and formalization of students’ algebraic and geometric work.

**Page 12.2-12.5**

Goals/Rationale

Students develop the traditional way to find the solution to a linear system, whether the eigenvalues are real or complex. This is important for those students who will be taking linear algebra and for many engineering fields where they are expected to use eigentheory regularly. The big point is the power of knowing just the eigenvalues as telling you everything of what you need to know to understand the phase plane. This is very powerful as well as more efficient for students going on in math and other STEM areas.

Implementation Notes and Student Thinking

This section will take two 75 minute class periods, most likely. Again, in this section, the teacher may choose to present this material in whatever way works for her. One idea is a flipped approach where the students watch a video you make outside of class and then work on problems in class. Alternatively, interactive lecture, or working as usual in groups are all appropriate. Instructors will need to scaffold at some point.

The purpose of these tasks is to develop and illustrate a more algebraically efficient method (at least some view it more efficient) for determining solutions to linear homogeneous systems of differential equations. Rather than first finding the slope of any straight line solutions (eigenvectors), students are introduced to a method for first finding the exponents for the straight line solutions (eigenvalues). Thus the name, eigenvalue first method.

Note that at the end of Page 12.3, an important connection is made to the use of determinants to find when two homogeneous linear equations are dependent. In comparison to work in the first activity with equilibrium solutions on Page 12.2 (students considered the equations ax + by = 0 and cx + dy = 0), we are now considering equations involving . It will be necessary for the teacher to help students see that (and why) we actually want (or force) these equations to be dependent. This is a difficult conceptual idea for students.

*Problem 6 -* There aretwo different ways of justifying the shape of graph that students might present: 1) the relative contribution of the two components and 2) the formal limit of *y(t)/x(t)*.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Homework**

*Problems 1-3* - Without using technology, students are expected to figure out the basic shape of graphs in the phase plane based on the eigenvalue and eigenvector information. For example, in problem 1, the phase plane is a sink because both eigenvalues are negative (all solutions are exponential decay). But how exactly do solutions approach (0,0)? Do they come into the origin along one or the other straight line solutions and why? Similar discussions should take place for time going to negative infinity and for problems 2 and 3.

*Problems 4* – additional practice with the new technique of finding eigenvalues first and sketching phase portraits.

*Problem 7* - intended to help students reflect on and organize what they have learned about eigenvalues. Students typically come to appreciate the power and usefulness of just knowing eigenvalues. Although just knowing the eigenvalues doesn’t tell one everything about solutions, it does provide much overall, qualitative information.

**Notes for Personal Reflection Unit 12**